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Tuned liquid dampers for controlling earthquake response of structures

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SUMMARY

Numerical simulations of a single-degree-of-freedom (SDOF) structure, rigidly supporting a tuned liquid damper (TLD) and subjected to both real and artificially generated earthquake ground motions, show that a properly designed TLD can significantly reduce the structure's response to these motions. The TLD is a rigid, rectangular tank with shallow water in it. Its fundamental linear sloshing frequency is tuned to the structure's natural frequency. The TLD is more effective in reducing structural response as the ground excitation level increases. This is because it then dissipates more energy due to sloshing and wave breaking. A larger water-depth to tank-length ratio than previous studies suggested, which still falls within the constraint of shallow water theory, is shown to be more suitable for excitation levels expected in strong earthquake motions. A larger water-mass to structure-mass ratio is shown to be required for a TLD to remain equally effective as structural damping increases. Furthermore, the reduction in response is seen to be fairly insensitive to the bandwidth of the ground motion but is dependent on the structure's natural frequency relative to the significant ground frequencies. Finally, a practical approach is suggested for the design of a TLD to control earthquake response. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: earthquake response; tuned liquid dampers; vibration control; sloshing; energy dissipation; TLD design

INTRODUCTION

The definition of design earthquake motions is rapidly becoming more stringent everywhere. Consequently, traditional methods of earthquake-resistant design are losing their relevance, as they become economically unviable. Current research has, therefore, focused on the development of devices that control a structure's response to levels amenable to its economic design. Tuned liquid dampers (TLD) are energy-absorbing devices that have been proposed to control the dynamic response of structures. A TLD, which usually is a rigid tank with shallow water in it, is connected rigidly to a structure. Tuning the fundamental linear sloshing frequency of the TLD to

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the structure's natural frequency causes large amount of sloshing and wave breaking at the resonant frequencies of the combined TLD-structure system that dissipates a significant amount of energy.

The TLD has been shown to effectively control the wind response of structures [1–5]. The main advantages of this device are the ease of fabrication and installation, especially where space constraints exist, and minimal maintenance after installation, which makes the device very cost-effective.

The TLD should also be effective for controlling earthquake response of structures. In fact, by its inherent property of allowing sloshing to occur in any direction, a properly designed rectangular TLD should be an ideal control device for bi-directional ground motion. A few experimental studies [6–8] have shown that a TLD does reduce the responses of flexible structures subjected to base motions. However, these studies were either illustrative in nature [6], or restricted to essentially investigating the effect on the energy dissipation of wave-breaking in the TLD [7,8]. In a recent paper [9], empirical expressions have been developed for determining the properties of an equivalent non-linear tuned mass damper model that captures the behaviour of a TLD system under a variety of loading conditions. There has, however, yet been no comprehensive study that investigates the effectiveness of a TLD for seismic vibration control and gives guidelines for design of such a TLD. Such a study [10] has recently been done for a tuned liquid column damper (TLCD), which is a variant of the TLD that dissipates energy by water flow between two water columns.

One of the objectives in this paper, therefore, is to study the effectiveness of a rectangular TLD in reducing the earthquake response of structures for various values of natural time periods and structural damping ratios. Furthermore, an attempt is made to define appropriate design parameters of the TLD that is effective in controlling the earthquake response of a structure. These parameters include the ratio of the linear sloshing and structure natural frequencies, henceforth called the tuning ratio, the ratio of the masses of water and structure, henceforth called the mass ratio, and the water depth to the TLD tank-length ratio, henceforth called the depth ratio. For numerical simulations, both actual ground motions and sets of artificial ground motions (different sets defining different frequency content and bandwidth of the ground motions) are considered.

PROBLEM FORMULATION

Structure idealization

A single-degree-of-freedom (SDOF) structure with a TLD attached to it and subjected to a ground motion is shown in Figure 1(a). The equation of motion for this SDOF structure is

$$m_s\ddot{v}_s + c_s\dot{v}_s + k_s v_s = -m_s a_g + F \tag{1}$$

where m_s, k_s and c_s represent the mass, stiffness and damping of the structure, respectively. Moreover, v_s is the displacement of the structure relative to the ground, a_g is the ground acceleration and F denotes the shear force developed at the base of the TLD due to water sloshing. Equation (1) when normalized with respect to the structural mass is given as

$$\ddot{v}_s + 2\xi_s\omega_s\dot{v}_s + \omega_s^2 v_s = -a_g + \frac{F}{m_s} \tag{2}$$

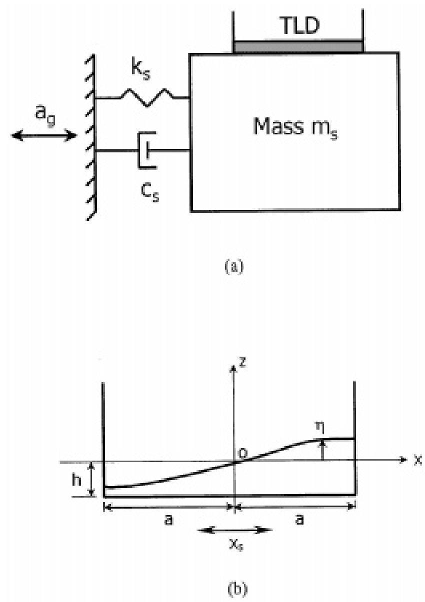

Figure 1. Schematic of (a) a single-degree-of-freedom structure with a rectangular tuned liquid damper and (b) dimensions of the rectangular tuned liquid damper.

where $\omega_s = 2\pi/T_s$ and ξ_s are the structure's natural frequency and damping ratio, respectively, and T_s is the natural time period. The TLD base shear force F, shown on the right-hand side of Equation (2), is determined by solving the equations of motion of the water in the TLD.

Formulation of TLD equations

The rigid rectangular TLD tank, which is shown in Figure 1(b), has a length $2a$ and width b (not shown in the figure), and an undisturbed water depth of h. It is subjected to a lateral base excitation, x_s, that is identical to the excitation of the structure's top. The equations of motion of the water inside the tank can be defined in terms of the free surface motion, as the water depth is assumed to be shallow [11]. Since strong earthquake ground motion generally results in large amplitude TLD excitation, the equations of motion should include the effects of wave breaking. The formulation used here has been suggested by Sun *et al.* [7], and the governing equations of motion of the water are

$$\frac{\partial \eta}{\partial t} + h\sigma \frac{\partial(\phi u)}{\partial x} = 0 \tag{3}$$

$$\frac{\partial u}{\partial t} + (1 - T_{\mathrm{H}}^2) u \frac{\partial u}{\partial x} + C_{\mathrm{fr}}^2 g \frac{\partial \eta}{\partial x} + gh\sigma\phi \frac{\partial^2 \eta}{\partial x^2} \frac{\partial \eta}{\partial x} = -C_{\mathrm{da}} \lambda u - \ddot{x}_s \tag{4}$$

where the independent variables are $\eta(x, t)$ and $u(x, \eta, t)$. They denote the free surface elevation above the undisturbed water level and the horizontal free surface water particle velocity, respectively. Both these variables are a function of the horizontal distance, x, from o [see Figure 1(b)]

and time t. The horizontal acceleration of the TLD base, which is identical to the total acceleration of the structure's top, is $\ddot{x}_s$, and the acceleration due to gravity is g. Equation (3) represents the integrated form of the continuity equation for the water, and Equation (4) is derived from the two-dimensional Navier–Stokes equation. The parameters σ, ϕ, and T_H in Equations (3) and (4) are given by the expressions [11,12]

$$\sigma = \tanh kh/kh \quad \phi = \tanh k(h+\eta)/\tanh kh \quad T_H = \tanh k(h+\eta) \tag{5}$$

where k is the wave number. The λ in Equation (4) is a damping parameter that accounts for the effects of the boundary layer along the tank bottom, side walls, and the water's free surface contamination that can be given semi-analytically [7,12] as

$$\lambda = \frac{1}{(\eta+h)}\frac{1}{\sqrt{2}}\sqrt{\omega_\ell v}\left[1+\left(\frac{2h}{b}\right)+s\right] \tag{6}$$

in which ω_ℓ is the fundamental linear sloshing frequency of the water in the tank, v denotes the kinematic viscosity of water, and s denotes a surface contamination factor which can be taken as unity [12]. The fundamental linear sloshing frequency of the TLD is given by [12]

$$\omega_\ell = \sqrt{\frac{\pi g}{2a}\tanh(\pi\Delta)} \tag{7}$$

where Δ is the ratio of undisturbed water depth h to the tank length $2a$, called the water depth ratio in this paper.

The coefficients C_{fr} and C_{da} in Equation (4) are incorporated to modify the water wave phase velocity and damping, respectively, when waves are unstable ($\eta > h$) and break [7]. These coefficients take on a unit value when waves do not break. Conversely, when waves break, C_{fr} is found empirically [7] to essentially have a constant value of 1.05, whereas C_{da} has a value that is dependent on the amplitude, $(x_s)_{max}$, of motion of the structure's top when it does not have a TLD attached to it. This C_{da} value is given as [7]

$$C_{da} = 0.57\sqrt{\frac{h^2\omega_\ell}{av}(x_s)_{max}} \tag{8}$$

where, as before, h and a are the water depth and half tank length, respectively, and ω_ℓ is the sloshing frequency given by Equation (7).

By solving Equations (3) and (4) simultaneously for the free surface elevation η, and neglecting higher-order terms and shear stresses along the bottom of the tank, a reasonable estimate of the shear force, F, at the base of the TLD is given by the following expression [12]:

$$F = \frac{\rho g b}{2}[(\eta_n+h)^2-(\eta_0+h)^2] \tag{9}$$

where ρ is the mass density of water, b is the tank width, and η_n and η_0 are the free surface elevations at the right and left walls, respectively, of the tank.

Analysis procedure

Equations (2)–(4) have to be solved simultaneously to find the response of a SDOF structure with a TLD attached. Although the structure's behaviour is linear, the water motion is non-linear. Therefore, an iterative numerical procedure is needed to compute the structure's response. Equations (3) and (4) are discretized, with respect to x, into difference equations and then they are solved using the Runge–Kutta–Gill procedure [13]. Equation (2) is solved using a central difference scheme in which the time step depends on defining the sloshing phenomenon properly but is also small enough to ensure numerical stability.

Definition of ground motions

A ground motion is typically characterized by its intensity and frequency content. Three types of ground motions are considered here: harmonic, recorded earthquake motions, and artificially generated earthquake motions.

The harmonic ground motion is only considered for defining the depth ratio of the TLD in relation to data published earlier. The excitation frequency ω and the amplitude of motion define this base excitation. Two excitation levels are considered here. One is a high excitation level that produces a relative displacement peak value at the top of the structure that corresponds to the one produced by the NS component of the 1940 El Centro ground motion. The other is a low excitation level corresponding to 10 per cent of the high excitation level and approximately similar to the excitation levels considered in earlier studies [6,11,13].

Three ground motions recorded at different sites for different earthquakes are also considered here. These motions are distinguished by their different frequency contents and intensity levels, as evident from their pseudo-acceleration response spectra given in Figure 2. The first one is the NS component of the El Centro ground motion, Imperial Valley 1940 earthquake, which has a peak acceleration of $0.348g$. The second one is the S69E component of the Taft ground motion, Kern County 1952 earthquake, which has a peak acceleration of $0.179g$. Finally, the third one is the NS component of the SCT ground motion, Mexico City 1985 earthquake, which has a peak acceleration of $0.101g$.

There is, however, a difficulty, which is discussed later in this paper, with studying the effectiveness of a TLD for recorded ground motions, which typically have uneven response spectra. This can be overcome by evening out the response spectra over a frequency spectrum. One approach is to take many recorded accelerograms, each normalized to an identical value of peak ground acceleration, and consider the mean structural response, as done, for example, in Reference [10]. Another approach, and the one chosen in this study, is to take a significant number of spectrum-compatible, artificially generated accelerograms and consider the mean structural response. The advantage of the second approach is that the frequency content and bandwidth of the earthquake ground motion can be controlled.

In this study, 20 artificial accelerograms are generated, using the software *PSEQGN* [14], from the time-modulated Kanai–Tajimi spectrum by defining particular values for its frequency parameter ω_g and damping parameter ξ_g. The time modulation function chosen is as given in Reference [14], with a parabolic rising function from 0 to 4 s, constant value of unity from 4 to 15 s, and an exponentially decaying function from 15 s to the final duration of 30 s. Three different sets are generated using three different Kanai–Tajimi filters, corresponding to a base white-noise ground motion filtered through hard (or shallow), medium, and soft (or deep) soil layers, respectively [15]. The set of motions filtered through hard soil has a frequency content centred at

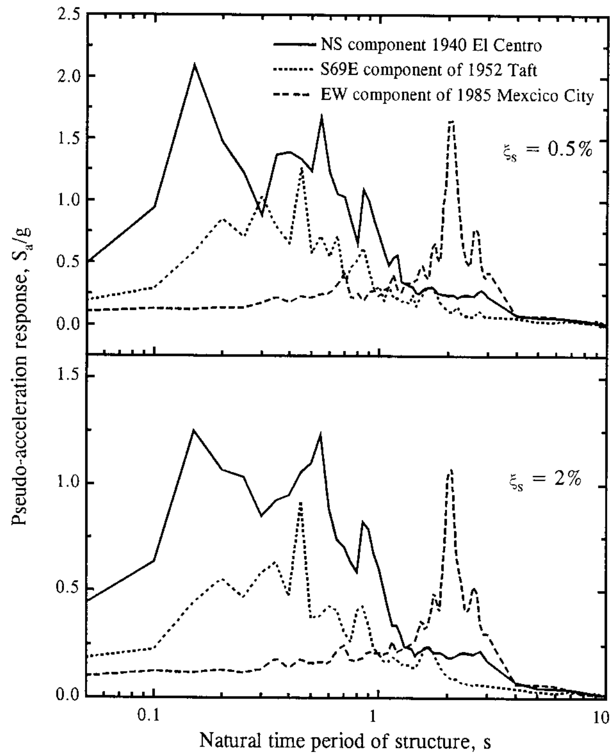

Figure 2. Pseudo-acceleration response spectra for recorded ground motions.

2.5 Hz ($\omega_g = 5\pi$ rad/s) with a large bandwidth ($\xi_g = 0.6$). The set filtered through medium soil has a frequency content centred at 1.5 Hz ($\omega_g = 3\pi$ rad/s) with a medium bandwidth ($\xi_g = 0.4$). The set filtered through soft soil has a frequency content centred at 0.5 Hz ($\omega_g = \pi$ rad/s) with a small bandwidth ($\xi_g = 0.2$). Each set, however, has the same mean peak ground acceleration of $0.35g$, approximately corresponding to the previously defined El Centro motion. The mean pseudo-acceleration response spectra for the three sets of ground motion are given in Figure 3. Note the relative smoothness of the response spectra.

SELECTION OF TLD PARAMETERS

The response of a structure with a TLD attached and subjected to a base excitation will depend on the characteristics of the structural-TLD system. It is obvious from Equation (2) that the structure is characterized by its natural time period, T_s, and damping ratio, ξ_s. The TLD has been shown to be characterized [5,13] by the three parameters defined earlier — the tuning, mass and depth ratios. A TLD may then be considered to be properly designed, if that TLD significantly reduces a structure's response to a design earthquake ground motion for a given set of values of the above parameters.

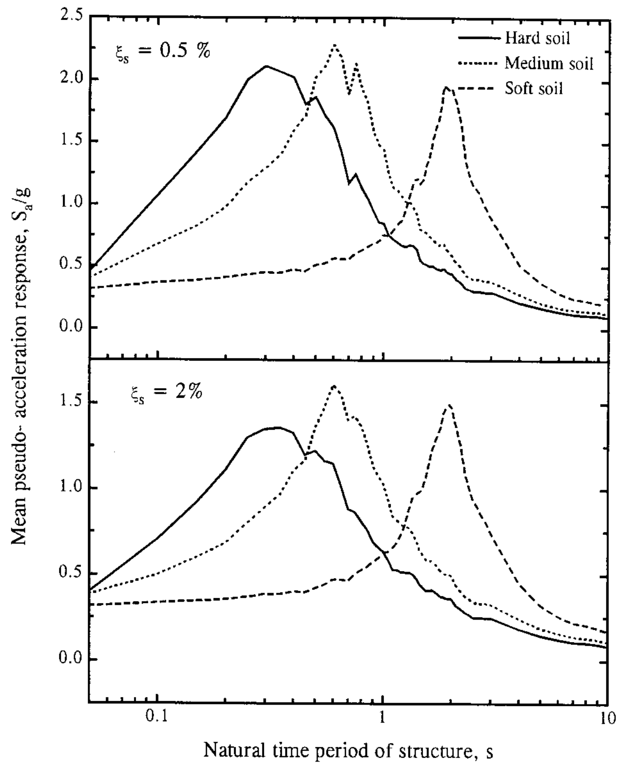

Figure 3. Mean pseudo-acceleration response spectra for artificial ground motions.

Tuning ratio

The tuning ratio, β, of a rectangular TLD, as defined earlier, is the ratio of the fundamental, linear sloshing frequency, ω_ℓ, which is given by Equation (7), to the natural vibration frequency of the structure, ω_s, or $\beta = \omega_\ell/\omega_s$. By convention, a tuned liquid damper implies that this tuning ratio is unity. Earlier experimental studies [7,13] have, however, shown that the optimum tuning ratio may depend on many factors, although it is mostly close to unity. Several numerical simulations by the authors for mass ratios less than 4 per cent, whose results are not shown for brevity, lead to the conclusion that it is reasonable to consider this tuning ratio to have a value of unity for strong earthquake motions also.

A comparison of the time histories of both the relative displacement and total acceleration at the top of a representative structure, with and without a TLD with a tuning ratio of unity, subjected to a typical earthquake ground motion is presented in Figure 4. This provides a good illustration of the manner in which a TLD reduces the peak seismic response of a structure. Firstly, note the slight elongation in the structure's time period as the TLD shifts the structure frequency. This fact will be important when the response of structures with a TLD attached are studied for recorded ground motions. Secondly, it can be seen that the TLD is not effective in the initial phase of the structure's vibration, because the water motion is then weak. Once the

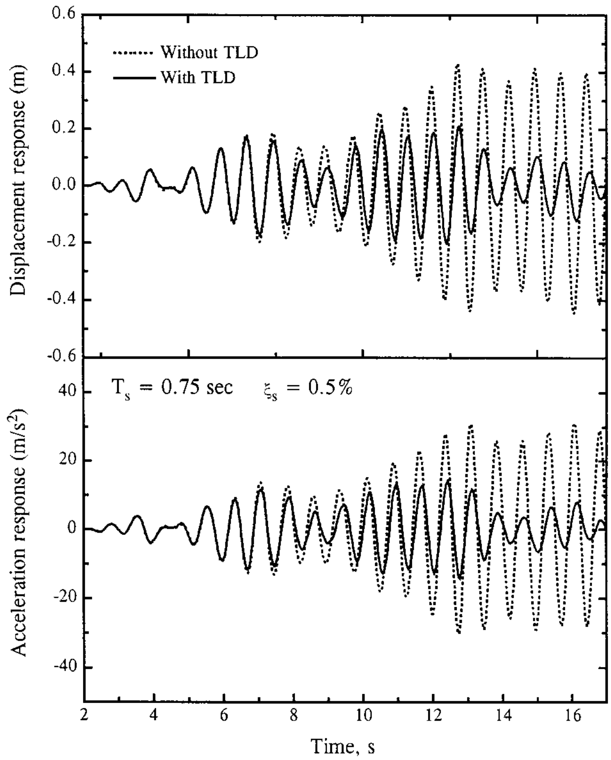

Figure 4. Structural response histories of a typical structure, with and without a LTD, when subjected to a typical ground motion.

strong motion phase starts, the TLD becomes increasingly effective in reducing the response, as water sloshing increasingly dissipates more energy. This leads to the conclusion, that a TLD may not be very effective in reducing the peak response of a structure subjected to a pulse-type ground motion. This is because for this type of motion the peak value is reached in the first couple of cycles of vibration, when the water motion does not get a chance to dissipate enough energy. This conclusion, however, needs additional analysis for confirmation.

Depth ratio

The water depth ratio Δ, which is the ratio of water depth h to tank length $2a$, as defined earlier, is a significant parameter for defining the effectiveness of a rectangular TLD. However, earlier studies [6–9,11–13] have restricted this value to less than 0.1, although the limit of the shallow water assumption is slightly less than 0.2 [5]. This may be because these studies used data originally developed for low excitation levels [11–13].

Here three values of $\Delta = 0.05$, 0.1, and 0.15 are considered in the TLD design for various values of structure period ($T_s \geqslant 0.5$ s) and damping ratios ($\xi_s = 0.5$ and 2 per cent) and two excitation levels of harmonic motion. The water depths required for the short period structure ($T_s = 0.5$ s) and a tuning ratio of unity are too small (≈ 1 cm for $\Delta = 0.15$) for practical implementation. An

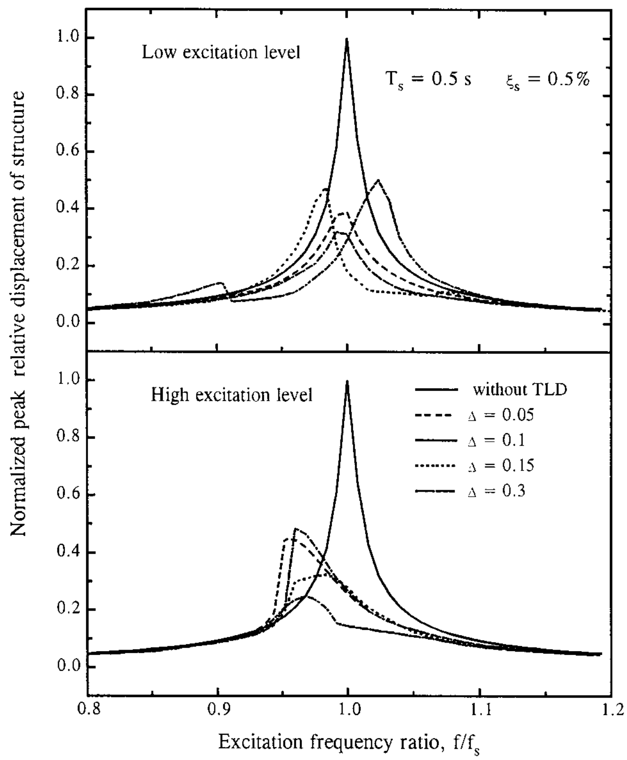

Figure 5. Frequency response functions for normalized relative displacement, for two excitation levels, of a short period structure ($T_s = 0.5$ s) with different water-depth ratios.

additional value of $\Delta = 0.3$ has, therefore, been considered for only that particular structure, although the corresponding water depth strictly violates the shallow water assumption.

The frequency response functions for the peak structural displacements, normalized with respect to the corresponding resonant peak value for the structure without a TLD whose mass ratio is 1 per cent, for two representative structures, with $\xi_s = 0.5$ per cent, are presented in Figures 5 and 6. The frequency response functions for all other parameter combinations show similar trends and are, therefore, not presented. Note the shift in the resonant frequencies of the system, which are greater for the high excitation level.

For the low excitation level, the resonant peak structural response typically increases with increasing depth ratio. This fact is also highlighted in Table I, where the maximum reduction in resonant peak structural response, for the low excitation level, is seen to occur for a depth ratio of 0.05, except for the short period structure for which the corresponding depth ratio value is 0.1. This is consistent with the depth ratios used in the earlier studies. For the high level excitation, however, the results presented in Table I lead to a different conclusion. The optimum Δ value is seen to shift to the highest values considered in the study, i.e. 0.3 for the short period structure, and 0.15 for the other structures.

The reason for this trend can be traced to the behaviour of a TLD. The energy absorbed and dissipated by the TLD depends mostly on sloshing and wave breaking. For a low excitation level,

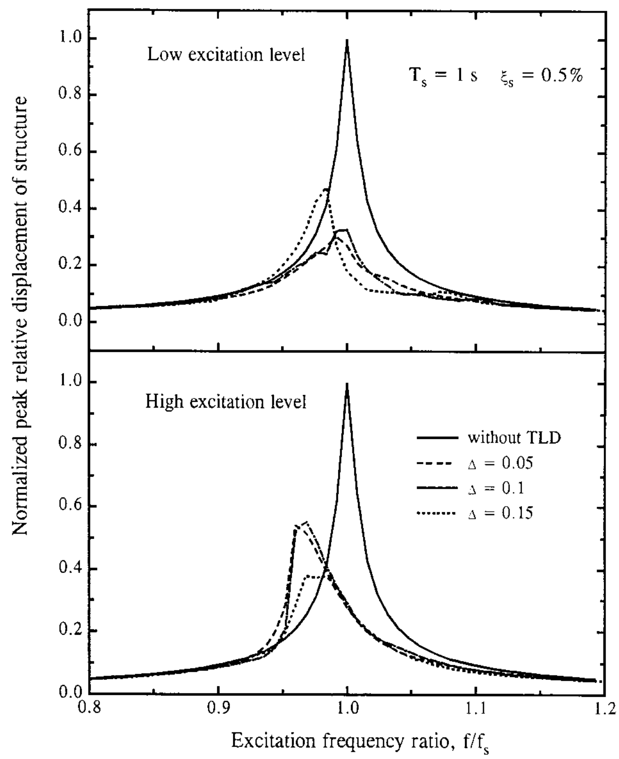

Figure 6. Frequency response functions for normalized relative displacement, for two excitation levels, of a medium period structure ($T_s = 1$ s) with different water-depth ratios.

the water in a TLD having a larger water depth, i.e. higher Δ value, does not slosh as much as that for a low water depth, because the energy transmitted then to the TLD is relatively low. For a high excitation level, however, the energy transmitted is large and even a TLD with relatively larger water depths dissipates energy through sloshing and wave breaking. For strong earthquake motions, therefore, it is more effective to use larger water depths, within the constraints of shallow water theory. From the results in Table I, it is interesting to note that a TLD beneficially reduces the resonant structural response more for a higher level of excitation. This is because, for these excitation levels there is more sloshing and wave breaking that increases the TLD's energy absorption and dissipation capacity. Furthermore, a TLD is actually more effective in reducing the structural response as the structure's period shortens. This implies that a TLD is more effective for shorter period structures. This is something that is also evident from the results for earthquake motions.

Mass ratio

The mass ratio, μ, which is the ratio of the mass of water to the structure's mass, as defined earlier, is also an important parameter to be considered in the TLD design. Intuitively, it seems that the

Table I. Percentage reduction in the resonant peak structural responses by a TLD, with 1% mass ratio and different depth ratios, for structures with $\xi_s = 0.5\%$.

Water depth ratio Δ	Percent red. for low excitation level			Percent red. for high excitation level		
	T_s (s)			T_s (s)		
	0.5	1	2	0.5	1	2
(a) *Total acceleration*						
0.05	61.2	70.0	64.9	59.3	50.4	30.6
0.1	67.9	67.3	47.4	55.6	48.5	32.0
0.15	61.6	47.3	41.7	68.5	63.4	57.8
0.3	46.6	—	—	76.4	—	—
(b) *Relative displacement*						
0.05	61.3	69.8	65.2	55.1	45.9	26.7
0.1	68.1	67.5	46.0	51.6	44.4	29.6
0.15	52.1	46.6	36.8	68.0	62.1	56.8
0.3	49.6	—	—	75.1	—	—

structural response should decrease with increasing mass ratio, as long as the water mass remains small in relation to the structure mass. This is because the larger volume of water for a higher mass ratio should absorb and dissipate more energy, without contributing significantly to the overall inertia of the system. From a practical viewpoint, however, a higher water mass implies a greater space requirement to install the TLD tanks on the structure, which may not always be feasible. In this study, three values of the mass ratio, i.e. 1, 2, and 4 per cent, are considered, which are small enough to be generally practical, yet large enough for the TLD to be effective as a control device.

ANALYSIS OF NUMERICAL RESULTS

Response to recorded ground motions

The peak structural responses, due to the recorded ground motions described earlier, and the reduction in them by a TLD are presented in Tables II and III. In the tables, $(a_{\max})_0$ and $(v_{\max})_0$ are the peak total acceleration and relative displacement values, respectively, at the top of the structure without a TLD. The results are presented for structures with natural periods across the spectrum from 0.5 to 2 s and damping values of 0.5 and 2 per cent, and a TLD mass ratio of 1 per cent. The results for other mass ratios do not provide any additional insight into the effectiveness of a TLD and are not presented here. The depth ratio for the 0.5 s natural period structure is taken to be 0.3, and for all other structures it is taken to be 0.15.

From these results it can be seen that a TLD provides comparable reductions in both the peak total acceleration and relative displacement of a particular structure. Furthermore, comparing the results in Tables II and III, it can be seen that the TLD chosen, with a relatively large depth ratio, is generally equally effective for the stronger El Centro ground motion as for the weaker Taft ground motion, which have similar ground frequency contents. This implies that, irrespective

Table II. Peak total accelerations at top of various structures, due to recorded motions, and the percentage reduction in them by a TLD with 1 per cent mass ratio (water depth ratio $\Delta = 0.3$ for $T_s = 0.5$ s structure; $\Delta = 0.15$ for all other structures).

Structural properties T_s (s)	ξ_s (%)	El Centro $(a_{max})_0$ (m/s^2)	El Centro Percent reduct. in a_{max}	Taft $(a_{max})_0$ (m/s^2)	Taft Percent reduct. in a_{max}	Mexico City $(a_{max})_0$ (m/s^2)	Mexico City Percent reduct. in a_{max}
0.5	0.5	12.2	4.9	5.65	31.7	2.09	17.2
	2.0	10.8	2.5	3.68	1.9	1.56	−3.9
0.67	0.5	13.6	38.6	6.19	31.2	4.28	47.4
	2.0	7.62	11.7	4.10	20.2	2.46	21.5
0.75	0.5	8.18	28.2	4.71	41.8	1.92	12.5
	2.0	6.24	20.5	2.53	0.8	1.60	−9.4
1	0.5	7.03	9.7	3.05	17.7	2.21	2.7
	2.0	5.99	9.2	2.07	6.8	2.12	9.4
1.5	0.5	2.45	7.4	2.12	12.7	4.33	19.6
	2.0	2.11	2.8	1.61	1.9	3.13	9.9
2	0.5	2.28	12.7	1.16	6.7	16.2	38.9
	2.0	1.87	12.3	1.04	6.7	10.2	17.2

Table III. Peak relative displacements at top of various structures, due to recorded motions, and the percentage reduction in them by a TLD with 1 per cent mass ratio (water depth ratio $\Delta = 0.3$ for $T_s = 0.5$ s structure; $\Delta = 0.15$ for all other structures).

Structural properties T_s (s)	ξ_s (%)	El Centro $(v_{max})_0$ (mm)	El Centro Percent reduct. in v_{max}	Taft $(v_{max})_0$ (mm)	Taft Percent reduct. in v_{max}	Mexico City $(v_{max})_0$ (mm)	Mexico City Percent reduct. in v_{max}
0.5	0.5	76.9	4.9	35.8	31.8	13.2	16.7
	2.0	68.2	2.5	23.3	2.2	9.8	−5.1
0.67	0.5	153	36.3	69.7	28.1	48.2	44.4
	2.0	85.8	8.6	46.0	17.6	27.7	11.2
0.75	0.5	116	25.3	67.0	40.0	27.4	11.0
	2.0	89.8	17.8	36.1	0.0	22.8	−9.2
1	0.5	178	9.1	77.4	18.2	56.2	−2.5
	2.0	152	8.7	52.5	8.0	53.6	8.8
1.5	0.5	140	7.0	121	14.3	246	17.8
	2.0	120	−0.6	91.9	4.2	178	5.3
2	0.5	231	12.5	118	7.7	1643	34.1
	2.0	190	12.3	105	7.7	1036	12.1

of actual ground intensities, a large depth ratio is preferable for all broad-banded strong earthquake motion.

The results presented in Tables II and III provide another insight into the behaviour of a TLD when attached to a structure. The effectiveness of a TLD with a fixed mass ratio to reduce peak structural response decreases with increasing structural damping. Similar trends are noticed across the spectrum of structures natural periods. This behaviour can be explained simply.

A TLD absorbs and dissipates energy based on the level of sloshing and wave breaking and, irrespective of the structural damping, this energy dissipation remains the same for a particular excitation level. For a structure having a larger structural damping, the added damping provided by the energy dissipation in the TLD decreases as a fraction of the overall damping. Thus, the effectiveness of the TLD is reduced.

Whenever the total acceleration is high at the top of a structure without TLD, e.g. note the 0.67 s period, 0.5 per cent damping structure for the El Centro motion and the 2 s period, 0.5 per cent damping structure for Mexico City motion in Table II, the TLD is effective in reducing the response. However, this reduction is not consistent across the structure natural period spectrum. This particular behaviour can be explained by referring to the response spectra of the ground motions. The TLD not only absorbs energy transmitted from the structure, but it also slightly shifts the structure's frequency, as seen earlier. For a particular structure and ground motion, the TLD may shift the frequency from one for which the response is low to one for which the response is high because the response spectra varies wildly for small changes in frequency. Thus, despite the added damping due to sloshing and wave breaking, the structural response with a TLD may not be substantially lower than that without a TLD. In fact, the response with a TLD may actually be slightly greater than without a TLD, as evident for the 0.5 s period, 2 per cent damping structure and the 0.75 s period, 2 per cent damping structure for Mexico City ground motion in Table II.

The unevenness of the recorded ground motion response spectra does not, therefore, provide adequate understanding of the effectiveness of a TLD in reducing earthquake response. It is for this reason that the mean structural response to a set of artificially generated earthquake motions, which have relatively smooth mean response spectra, is studied next.

Response to artificial ground motions

A comparison of the mean peak responses of structures, with and without a TLD, for artificial motions corresponding to hard, medium and soft soil conditions are presented in Tables IV–VI, respectively. Note that the structural and TLD parameters considered and notation for structural responses are the same as those for recorded ground motions. In addition, the comparison of TLDs with different mass ratios has been presented. It can be seen immediately that the trends become much more discernible than do those for the recorded ground motions. The maximum reduction in the structural responses provided by the TLD corresponds to the cases where the structural frequency is in the vicinity of the significant ground frequency and the structural response without a TLD is large. Since the reduction in responses for the different frequency bandwidth motions in Tables IV–VI are generally comparable, it is implied that a properly designed TLD is equally effective for a broad banded ground motion as it is for a narrow banded motion. Thus, one can conclude that the effectiveness of a TLD is fairly insensitive to the frequency bandwidth of ground motion.

It can also be seen that, for a given structural damping and period, increasing the mass ratio from 1 to 4 per cent makes the TLD more effective in reducing structural response. This confirms the premise made earlier in this paper. Furthermore, the response reduction provided by a TLD with 1 per cent mass ratio for a structure with 0.5 per cent damping is equivalent to that provided by a TLD with 4 per cent mass ratio for a structure with 2 per cent damping. This implies that a structure with higher intrinsic damping requires a TLD with a larger mass ratio to provide similar reduction to that needed for a structure with lower damping. The reason can be

Table IV. Mean peak responses at the top of various structures, due to artificial motion on hard soil, and the percentage reduction in them by a TLD of varying mass ratio (water depth ratio $\Delta = 0.3$ for $T_s = 0.5$ s structure; $\Delta = 0.15$ for all other structures).

Structural properties		$(a_{max})_0$ (m/s^2)	Percentage reduction in a_{max}			$(v_{max})_0$ (mm)	Percentage reduction in v_{max}		
T_s (s)	ξ_s (%)		$\mu = 1\%$	$\mu = 2\%$	$\mu = 4\%$		$\mu = 1\%$	$\mu = 2\%$	$\mu = 4\%$
0.5	0.5	18.3	31.8	45.4	51.3	117	31.6	44.9	49.5
	2.0	12.2	16.2	23.8	33.4	77	15.6	22.3	30.5
0.67	0.5	12.6	29.8	34.1	39.7	142	26.1	30.1	33.4
	2.0	9.30	16.6	22.0	27.7	99	8.1	12.5	16.3
0.75	0.5	12.3	29.9	36.1	45.0	175	26.3	31.6	39.0
	2.0	8.53	14.0	21.1	29.2	121	9.9	16.3	23.0
1	0.5	8.33	22.9	31.8	40.2	211	20.3	27.1	35.0
	2.0	6.25	13.1	19.7	28.1	158	10.0	15.7	23.7
1.5	0.5	5.19	17.2	25.8	33.7	296	15.1	22.8	30.5
	2.0	4.19	11.5	17.7	24.6	239	9.7	15.1	22.1
2	0.5	4.35	14.3	23.2	33.2	441	12.7	21.6	31.9
	2.0	3.52	9.9	16.4	26.2	357	8.9	15.4	24.7

Table V. Mean peak responses at the top of various structures, due to artificial motion on medium soil, and the percentage reduction in them by a TLD of varying mass ratio (water depth ratio $\Delta = 0.3$ for $T_s = 0.5$ s structure; $\Delta = 0.15$ for all other structures).

Structural properties		$(a_{max})_0$ (m/s^2)	Percentage reduction in a_{max}			$(v_{max})_0$ (mm)	Percentage reduction in v_{max}		
T_s (s)	ξ_s (%)		$\mu = 1\%$	$\mu = 2\%$	$\mu = 4\%$		$\mu = 1\%$	$\mu = 2\%$	$\mu = 4\%$
0.5	0.5	20.0	34.0	42.7	48.4	127	31.5	41.4	44.7
	2.0	13.5	16.7	24.3	28.4	85	15.3	22.1	23.3
0.67	0.5	19.9	37.8	37.0	41.0	225	34.7	32.5	32.3
	2.0	14.6	21.8	23.5	26.9	164	17.1	18.3	17.7
0.75	0.5	20.6	40.1	42.6	48.8	294	36.7	38.9	42.5
	2.0	14.1	21.8	24.3	32.1	200	22.0	19.5	19.5
1	0.5	13.6	28.7	36.3	44.6	344	25.7	31.6	37.6
	2.0	10.2	16.6	24.2	33.0	258	13.3	18.8	25.8
1.5	0.5	7.49	17.9	25.6	32.3	429	15.3	22.7	28.2
	2.0	6.07	11.5	17.5	24.5	346	8.6	14.0	19.6
2	0.5	6.00	15.2	24.8	34.9	608	12.7	21.4	30.6
	2.0	4.86	10.7	18.4	26.2	493	8.9	15.4	23.0

traced to the relative energy dissipation capacities of the structure and TLD, as already mentioned earlier.

Finally, it can be concluded that a TLD is indeed effective in reducing the structural response to earthquake motion. For example, a TLD with 4 per cent mass ratio attached to a structure with 0.5 s time period, subjected to a high-frequency broad-banded earthquake motion, reduces the

Table VI. Mean peak responses at the top of various structures, due to artificial motion on soft soil, and the percentage reduction in them by a TLD of varying mass ratio (water depth ratio $\Delta = 0.3$ for $T_s = 0.5$ s structure; $\Delta = 0.15$ for all other structures).

Structural properties		$(a_{\max})_0$ (m/s^2)	Percentage reduction in $a_{\max}$			$(v_{\max})_0$ (mm)	Percentage reduction in $v_{\max}$		
T_s (s)	ξ_s (%)		$\mu = 1\%$	$\mu = 2\%$	$\mu = 4\%$		$\mu = 1\%$	$\mu = 2\%$	$\mu = 4\%$
0.5	0.5	5.09	18.7	22.3	22.8	32	17.7	20.9	20.2
	2.0	4.15	6.8	9.1	7.3	26	5.9	7.3	4.4
0.67	0.5	5.34	14.4	16.7	17.2	60	12.4	13.8	12.4
	2.0	4.47	5.4	6.8	6.9	50	3.8	4.1	2.1
0.75	0.5	5.98	12.6	17.3	19.1	85	10.2	13.8	13.9
	2.0	4.97	5.7	7.6	8.4	71	3.4	4.2	3.2
1	0.5	7.46	14.1	19.3	22.6	189	11.5	15.2	16.3
	2.0	6.17	7.6	10.4	12.1	156	5.0	6.3	6.2
1.5	0.5	11.8	14.2	15.8	14.5	671	10.1	8.9	0.1
	2.0	9.89	7.3	8.8	7.7	563	3.3	1.5	−7.1
2	0.5	19.1	31.0	40.4	46.4	1933	28.1	35.5	39.4
	2.0	14.6	21.2	30.0	35.8	1480	18.6	24.5	28.8

structural response by approximately 50 per cent if the structural damping is 0.5 per cent, and by approximately 33 per cent if the structural damping is 2 per cent (see Table IV).

Design of a TLD

It is instructive and practically useful to review the design of a TLD that has been found to be effective in significantly reducing earthquake response of a SDOF structure. The suggested steps are as follows:

1. Determine the structure frequency, ω_s, and select the fundamental sloshing frequency, ω_ℓ, of the TLD to be equal to it, considering the tuning ratio to be unity.
2. Then compute the required tank length, $2a$, from Equation (7), considering the depth ratio, Δ, to be as close to the shallow water depth limit, say 0.15. Then compute the water depth, h, which is given as $2a\Delta$.
3. Finally compute the tank width, b, from Equation (7), where $\Delta = h/b$ and the fundamental sloshing frequency is tuned to the structure frequency in the other direction. For example, $b = 2a$ if the frequencies of the structure in the two orthogonal directions were identical. This is just to ensure that the TLD is equally effective for ground motion in the orthogonal direction too.
4. Compute the mass of water in one tank, m_l, from the expression $m_l = 2\rho hab$, where ρ is the mass density of water. Considering the mass ratio μ to be 4 per cent, compute the N number of tanks required from $N = \mu m_s/m_\ell$, where m_s is the structure mass.
5. Compare the floor area available at the top of the structure with that required by the N number of tanks. If the area available is less, compute the number of tanks that would fit in the available area, and compute the available mass ratio. As a rule of thumb, if the available mass ratio is less than 1 per cent, and the structure damping is more than 2 per cent, then the TLD is not going to be effective as a control device.

SUMMARY AND CONCLUSIONS

The emphasis in this paper is to study the effectiveness of a rectangular TLD in controlling response of structures subjected to earthquake ground motions, and to define the design parameters of this TLD: tuning ratio, depth ratio, and mass ratio. In fact, steps are given for an efficient design of a TLD for controlling earthquake response of structures.

From numerical simulations it is shown that a higher depth ratio of 0.15 for a TLD, which is closer to the limit of the shallow water assumption, is more effective for high excitation amplitudes expected in strong earthquake ground motions. It is also shown that the effectiveness of a TLD increases with increasing structural base excitation level, as energy dissipation due to sloshing increases, and reduces as structural damping increases, as the relative energy dissipation by the TLD decreases.

The analysis of various SDOF subjected both recorded and artificially generated ground motions shows that a properly designed TLD with a low mass ratio of 1 to 4 per cent can substantially reduce structural response. A larger value of mass ratio induces a greater reduction in the structural response, and is preferable if the structure has a high intrinsic damping. The TLD becomes more effective in reducing structural response as the natural period of the structure shortens.

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